the absence of any authoritative information on the subject, we may reasonably conjecture that it signifies French (or Royal) America, and, if this be so, the only characteristic needed to make this coin pre-eminent in interest among all American colonial pieces, that, namely, of having on its face a distinct mention of our Continent, is supplied.

In concluding this paper, of which the subject, and the treatment of the subject, will, it is hoped, not be found beneath the notice of the American Philosophical Society, the writer may be permitted to observe that the fact of its being presented in this our year of Jubilee, and at the moment whence, a century ago, our first grand "annorum series" began to proceed, is entirely accidental, resulting from the casual acquisition, at this time, of the specimen which accompanies it. There seems, however, to be an eminent propriety in calling to mind, on the present most interesting occasion, and also connecting with tangible objects of curiosity, however slight, the American history of the great ally who rendered such essential aid to the insurgent colonies during their doubtful struggle. In regard to matters like this, perhaps not likely to be remembered with sufficient tenacity, medals and coins perform an important service; while, to take a broader view, through their distinct marking of decisive epochs, they contribute to enable us, in the words of a writer of the illustrious nation referred to, "*vivre de la grande vie des siècles*"—to live in the great life of the Centuries.

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*On some Fundam ntal Propositions of Central Force.**

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(*Read before the American Philosophical Society, July* 21, 1876.)

All cyclical motions which are produced by the action of a central force are, necessarily, of an oscillatory character. They may, therefore, be mathematically represented by suitable modifications of simple pendulum equations, as Fourier has well shown.† The first attempt at a general discussion of such motions seems to have been made in 1827, by Dr. Henry James Anderson, Professor of Mathematics and Astronomy in Columbia College, whose paper‡ may be found in the third volume of the second series of the

* Whenever I speak of "central force," in the present paper, I refer to force varying inversely as the square of the distance.

† See, also, papers by David Rittenhouse, Trans. Soc. Phil. Am., iii; Jos. Clay Ib., v; Owen Nulty (suggested by Rittenhouse's), Ib. [2] 1; James Dean, and N. Bowditch, Trans. A. A. S., iii; Robert Adrain, Trans. S. P. A., [2] i; Eugenius Nulty, Ib. [2] ii.

‡ "On the motion of Solids on Surfaces, in the two Hypotheses of perfect Sliding and perfect Rolling, with a particular examination of their small Oscillatory Motions." *Op. cit.* p. 315.

American Philosophical Transactions. The following quotation from his exordium is both prophetic and suggestive :

"There are few branches of Mechanical Philosophy as interesting in every point of view as the theory of Oscillatory Motion. From the minutest vibrations of a harp-string to the magnificent oscillations of a planet's axis, there are an infinite number of analogous phenomena remarkable for their curious properties or important uses. The common pendulum, that little instrument which has rendered such essential service to science and the arts, and will soon, in the hands of the skilful observer, unfold to us the internal constitution of our globe, and give a clue to the process by which it has acquired its present state, is itself indebted for its accuracy to the incessant superintendence of a watchful mathematical analysis. The science of Acoustics in all its parts, the varied phenomena of the tides, the theory of Saturn's ring, that wonder of the solar system, and the philosophical explanations of the stability and harmony of the celestial motions, are in fact, but different applications of this extensive branch of Demonstrative Mechanics. What adds to the interest and value of this subject is the circumstance that a large class of oscillatory motions, namely those of any rigid system whatever whose points depart but little from the position which they occupy when at rest, has been found susceptible of complete determination, by means of which the position of the bodies composing the system may be expressed (to use the language of analysis) in finite functions of the time."

In 1843, Professor Stephen Alexander communicated to the American Philosophical Society,* his observations upon physical phenomena which accompany eclipses. Among those phenomena was a "dragging of shadows," which he attributed, at the first meeting of the American Association (Philadelphia, 1848), to the inertia of the luminiferous æther. By this, and other like "scientific uses of the imagination," he was led to the discovery of a series of cosmical relations, some of which were laid before subsequent meetings of the Association, the whole being finally embodied in his "Statement and Exposition of Certain Harmonies of the Solar System."†

At the second meeting of the American Association, (Cambridge, 1849), Professor Benjamin Peirce read a paper "On the Relation between the Elastic Curve and the Motion of the Pendulum." "On this subject Prof. Peirce remarked, that the relations discovered merely by intellectual investigations, and not observed by the senses, are of peculiar interest, as manifesting the fact that one intellect presides over the production of those phenomena. Could we see in the moon a house like our own, we should say that it was built by men like ourselves, having similar wants, and using similar means to supply them, and we should say that the same being who formed our minds created theirs also. We cannot make such observations, but we may trace relations between objects with which we are

* Proc. Soc. Phil. Amer., May, 1843, vol. iii.

† Amer. Nat. Acad. Sci., 1873-4; Smithsonian Contrib., 280.

familiar, which lead us to similar results. At present the discovery of these relations has been very much confined to those subjects to which mathematics apply."* At a later session of the same meeting, Peirce presented a "Mathematical Investigation of the Fractions which occur in Phyllotaxis." After showing the influence of an identical law in the arrangement of plants and planets, he asks: "Whence could this extraordinary coincidence have arisen but from the action of a single mind? and what does it indicate but that the same Word which created the planet, is expressed in the planet? May I close with this remark, that the object of geometry in all its measuring and computing, is to ascertain with exactness the plan of the great Geometer, to penetrate the veil of material forms, and disclose the thoughts which lie beneath them? When our researches are successful, and when a generous and heaven-eyed inspiration has elevated us above humanity, and raised us triumphantly into the very presence, as it were, of the Divine intellect, how instantly and entirely are human pride and vanity repressed, and by a single glance at the glories of the infinite mind, we are humbled to the very dust."†

On the second of January, 1849, in a communication "On the Fundamental Principles of Mechanics,"‡ Peirce had already shown that "a system of bodies in motion must be regarded mechanically as a system of forces or powers which is a perfect representative of all the single powers of which the system is compounded, and this, too, at whatever time or times the component powers may have been introduced into the system. The question of the simultaneous introduction of the partial powers is of no importance. Any power which is at any time communicated to the system is preserved in the system unchanged in amount or direction."

At the same meeting of the Academy, Professor Joseph Lovering read a paper "On the Law of Continuity,"§ in which he said, "the method of analysis which began with Leibnitz and Newton, and which in England has been known under the name of fluxions, rests upon this law of continuity. If we admit the usefulness of the principle only in cases of motion, we still give it a wide range; since so many problems, not strictly dynamical, are reduced to cases of motion when investigated by the rules of modern analysis."

On the 4th of Feb. 1851,‖ "Professor Peirce gave an argument, which he thought to be new, against the principle which is usually adopted in theoretical works, that the force of a body in motion is its *vis inertiæ*. He believes, on the contrary, that the time is at hand when the *vis viva* will be universally recognized as the force of a moving body. His new argument is derived from the effect of a force in causing rotation, as well as translation. By the old theory, no additional force is required to produce rotation; whereas, by the theory of the *vis viva*, just as much force is re-

* Proc. A. A. A. S., ii, 129.

† Ib., p. 446.

‡ Proc. A. A. S., ii, 111.

§ Ib., p. 121.

‖ Ib., p. 256.

quired as is actually exhibited in the resulting rotation. The same argument may be derived in another form from the vibrations of elastic bodies."

The "Mathematical Monthly," for January, 1859, contained an exhaustive discussion of "The Motions of Fluids and Solids relative to the Earth's surface," by Professor William Ferrel, some of the results having been published about two years before, in the "Nashville Journal of Medicine and Surgery." The recent great advance in meteorological knowledge and in successful weather prediction is due, in great measure, to the publication of Ferrel's laws.

In 1861, Professor Simon Newcomb presented to the American Academy, a compendious review of the dynamical theory of gases.* His paper contains, among other matters of interest, the following first approximation towards a theoretical establishment of the empirical ratio between the temperatures under constancy of pressure and under constancy of volume : "7. If the particles were perfectly hard and spherical, the specific heat under constant volume would be to that under constant pressure as 3 to 5. If they were hard, but not spherical, this ratio would be that of 3 to 4. The latter result follows from an elegant theorem given by Professor Maxwell in Vol. XX, of the Philosophical Magazine, viz., that if the particles are hard, but not spherical, the sum of their *vires vivæ* of translation will be equal to that of their *vires vivæ* of rotation."

My first attempt at applying the foregoing principles was made in 1863,† when I showed that the daily fluctuations of the barometer are of such form and magnitude as can be explained by the combined action of atmospheric inertia, terrestrial resistance, and *motion* relative to the Sun. I also showed that the tendencies to equality of areas, in the daily and yearly fluctuations, furnish the data for a close approximation to the solar distance.‡

In 1864, Professor James Edward Oliver made a practical investigation of the best approximate representation of all the mutual ratios of *k* quantities by those of simple integers, drawing illustrations especially from phyllotaxis and planetotaxis. His investigation assumed that "Nature, while obeying with absolute precision the resultant of her efficient laws, such as Inertia, Attraction, etc., falls into forms which commonly both *utility* and *taste*, often independently of each other, would prescribe."§

In 1865, Ferrel described an annual variation in the daily mean level of the ocean, the greater part of which he attributed to "a tangential force arising from the motions of the ocean combined with the motion of the Earth's rotation. It was first brought out in its most general form in [his] paper on the motions of fluids and solids relative to the Earth's surface."‖

* Proc. A. A. S., v, 112.

† Proc. Soc. Phil., Amer., ix, 283.

‡ Ib., *loc. cit.*, and x, 376. See, also, Ferrel's paper of 1865, to which reference is made below.

§ Ib., vi, 288.

‖ Ib., vii, 35.

A few days ago, Dr. Thomas Hill, Ex President of Harvard College, illustrated the danger of exalting science above philosophy. "The true aim of science is not merely to record the uniformities of nature, but to discover the intellectual ideal which binds them together. The human mind in the presence of phenomena * * * is always inclined to ask three questions: What is the invariable order of sequence? What cause produces the effect? For what end are they produced?"*

Each of these papers involves one or more of the following postulates, which are treated by many as scientific heresies, but which others regard as prolific truths.

1. Faith and Reason are both handmaids of Science.

2. Knowledge is not only power, but it is also supreme power, or the source of all power.

3. Order and harmony are evidences of Intelligence. The discovery of new harmonies should, therefore, always stimulate new scientific investigation.

4. There can be no law without a Lawmaker.

5. Unity of Intelligence indicates unity of law.

6. Æsthetic gratification points to efficient laws.

7. All empirical results rest on *a priori* principles.

8. The methods, are permanently recorded in the works, of Intelligence.

9. There can be no unbalanced physical force without motion Even forces which are *relatively* static, can only be fully studied when producing motion.

10. Simple physical force is always central. Therefore the laws of elasticity and of oscillatory motion are of prime importance in all fundamental physical investigations.

11. Any æthereal medium, through which impulses are progressively transmitted, must be material.

12. Any medium through which impulses are transmitted instantaneously,† must be devoid of inertia, and therefore spiritual.

13. Tendency of motion is always in the direction of least resistance. Radial and tangential oscillations naturally alternate.

14. If the force of a moving body is its *vis viva*, the average *vis viva* of a perpetual oscillation is the proper standard for determining its quantity of motion, and the average velocity is the velocity of an equivalent synchronous oscillation under uniform velocity.

Most of these postulates, like all hypotheses, theories and laws, fall within the domain of metaphysics. Physical science, properly so called, is limited to facts which are discoverable by the senses, and to their co-ordination in accordance with metaphysical requirements. Pure metaphysics tends to

* Address before the Alumni of Bowdoin College, July 12, 1876.

† The only instance of supposed instantaneous transmission is that of gravitating action. La Place s estimate of its least possible velocity has been variously and sometimes wrongly stated; in his *Mécanique Céleste*, X, vii, 22, he fixes it at 100,000,000 times the velocity of light, and says, "therefore mathematicians may continue to regard it as infinite."

dreaminess and paradox ; pure physics, if such a thing were possible, to worldliness and shallowness. Metaphysics strives to learn too much ; physics is satisfied with too little. Science deals both with the physical and with the metaphysical ; embracing all knowledge that bears, in any way, upon the religious, the ethical, or the intellectual needs of man. An exclusive regard to either phase of our triune nature leads almost of necessity, to shortsightedness and narrowmindedness. In the modern science of thermodynamics, the fertility of discovery, which has rewarded the labors of European and American investigators, furnishes abundant evidence of the usefulness of metaphysical anticipations as aids to physical research.

Newton looked to ætherceal oscillation as a probable source of gravitation, finding it easier to imagine the approach of inert bodies through the influence of elastic thrusts, than to conceive of any practicable form of pull; Le Sage, adopting Newton's view, developed it with considerable detail, and later French Academicians have shown, in various ways, and to various extents, the likelihood of the hypothesis ;* Anderssohn has given an experimental demonstration of its sufficiency ; Norton and Challis have engaged in similar speculations, which they have extended so as to embrace all known forms of force ; Weber, Kohlrausch, and Clerk Maxwell have confirmed Faraday's partial identification of light and electricity by actual measurement ;† Faraday's cautious suggestion of "lines of force," proposed to treat all forces mathematically and independently of any hypotheses as to their nature, a proposal which has been largely and wisely followed by many recent investigators. As a further contribution towards such treatment I submit the following propositions, to which I have been led in the progress of my own researches, and which may be serviceable in other investigations.‡

Let $f =$ velocity communicable, at distance r in time t, by any central force varying inversely as r^2.

$f' =$ velocity communicated by a single impulse or in a single instant.

$f_i = if' =$ " " by i equal impulses, or in i instants if the distance remains constant. Then

I. If the pressure resulting from f is constant, it must either be exactly

* For some of the most important of these papers, see references of W. B. Taylor, in Journal of the Franklin Institute, cii, 71.

†The 9th postulate may, perhaps, be made still more general by this measurement. That we know no instance of absolutely static force, is indisputable ; the communication of momentum or *vis viva* by any physical agency which has no momentum nor *vis viva* is, to me, inconceivable ; the consideration of electrostatic force as force static relatively to the Earth, and of electrodynamic force as force moving with the velocity of light, furnishes almost demonstrative evidence of a universally oscillating æthereal medium which may be the secondary or physical source of all physical motion and all physical force.

‡ Most of the propositions of Newton's Principia are applicable to all central forces.

counterbalanced by some opposing force or forces, so as to produce relative rest ; or the motion must be maintained at a constant distance from the centre, so as to produce circular revolution ; or, if the opposing force results from the transformation of interfering revolutions, the two conditions may be combined so as to produce rotation. The velocity of circular revolution $= \sqrt{fr}$; the velocity of rotation $= \sqrt{fr} \div n$. The value of n may be found by experiment, or, if the aggregating impulses are homogeneous, it may be easily calculated, as will be subsequently shown (V–VII).

II. If the pressure is varying, and so exerted as to produce radial motion, directly towards the centre, the velocity acquired, at any given distance ρ, may be found by the equation $v_\rho = \left\{\frac{2fr(r-\rho)}{\rho}\right\}^{\frac{1}{2}}$. Therefore, if the theory of Boscovich were true, at the centre, where $\rho = 0$, v_ρ would be infinite.

III. If the varying pressure, or pull, leads to perpetual radial oscillations, synchronous with perpetual circular oscillations under constant pressure, or pull, the extent of the radial excursions from the centre is $2r$, and the mean velocity of radial oscillation (Postulate 14) is $\frac{2}{\pi}\sqrt{fr}$. The equation $\frac{2}{\pi}\sqrt{fr} = \left\{\frac{fr\,(2r-\rho)}{\rho}\right\}^{\frac{1}{2}*}$ gives $\rho = \frac{2\pi^2 r}{\pi^2+4} = 1.4232r$.

IV. Since the *vis viva* of a moving particle varies as $\frac{1}{r}$, the *vis viva* at the radius of average radial velocity in a rectilinear orbit, : the *vis viva* at the radius of synchronous circular velocity : : 1 : 1.4232.

V. If the constrained synchronous rotation of particles in a spheroid, and the free revolution of exterior particles, are due to the same primitive æthereal impulses (I), the uniform velocity of those impulses is the *limiting* velocity, towards which both motions tend when their circular paths are indefinitely diminished. Let, therefore, $v_1 = \sqrt{fr} \propto \sqrt{\frac{1}{r}}$ = velocity of free equatorial revolution ; $v_2 = \frac{\sqrt{fr}}{n} \propto \frac{1}{r}$ = constrained velocity at the same point = velocity of superficial equatorial rotation. Then $v_0 = \left[v_1\right] = \left[v_2\right] = nv_1 = n^2v_2 = n\sqrt{fr}$ = limiting tangential velocity, both of revolution and of rotation, under a reduction of spheroidal volume to an equatorial radius $\frac{r}{n^2}$. Under such reduction, all the particles in the equatorial plane would have the velocity of free revolution, or of perfect fluidity.

*Since $f \propto \frac{1}{r^2}$, the substitution of $2r$ for r_0 gives $fr\,(2r - \rho) = 2f_0r_0\,(r_0 - \rho)$.

VI. If contraction were to go still further, the centrifugal force of rotation would give the nucleal particles orbits of increasing ellipticity. These orbits would finally become infinite, when the average linear motion was equivalent to parabolic perifocal motion; or, in other words, when the mean linear velocity, $\frac{2}{\pi}\left[v_2\right] = \left[\sqrt{2fr}\right]$. These velocities tend to or from coincidence at $\frac{2r}{n^2\pi^2}$; for $\frac{n^2\pi^2}{2} \times \frac{2}{\pi} \times \frac{\sqrt{fr}}{n} = \frac{n\pi}{\sqrt{2}}\sqrt{2fr} = n\pi\sqrt{fr}$. We thus find that the limiting velocity between complete dissociation and incipient aggregation $(n\pi\sqrt{fr})$, is $\pi \times$ the limiting velocity between complete aggregation and incipient dissociation $(n\sqrt{fr}$; see V.)

VII. Let t = time of describing $\frac{1}{2}r$, in virtual approach to the centre. under the action of any central force f, = time of describing r in circular revolution, or motion under constant pressure. Then $ft = \sqrt{fr}$; πt = time of free semi-circular oscillation; $n\pi t$ = time of constrained semi-circular oscillation = time of semi-rotation; $n\pi ft = n\pi\sqrt{fr}$ = velocity acquired in the time of semi-rotation = *radial limiting* velocity (V, VI). Therefore the velocity of any central impulses which are capable of producing aggregation, free revolution, and constrained rotation = velocity produced by constant equatorial pressure acting for one half-rotation.

VIII. Radial oscillations, through a radius $\rho = mr$, give the central dissociating velocity $\left[\sqrt{2fr}\right]$ at $\frac{mr}{m+1}$.

IX. Undulations in elastic media tend to generate other undulations, in arithmetic, geometric, harmonic and other figurate progressions.

X. The limiting radius of free revolution (or the atmospheric radius in nebular condensation) varies as $t^{\frac{2}{3}}$; the limiting radius of constrained rotation (or the nucleal radius) varies at $t^{\frac{1}{2}}$. Therefore the nucleal radius $\propto \frac{3}{4}$ power of the atmospheric radius. In ordinary discussions of the nebular hypothesis, planetary aggregation has been regarded as atmospheric, under a velocity varying as $\sqrt{\frac{1}{r}}$; but there are many traceable evidences of simultaneous nucleal activity, under a velocity varying as $\frac{1}{r}$.

ILLUSTRATIONS.

Electrodynamics and thermodynamics furnish numerous illustrations of Prop. I, but some of its most obvious exemplifications are found in cosmical revolution, atmospheric elasticity, axial rotation, and in the various applications of Ferrel's laws.

Proposition II identifies all central forces, so far as an identical ideal limit of velocity is concerned. It is true that the ideal limit is physically unattainable, but a full development of Peirce's theory, of *vis viva* and simultaneous rotation and transmission,* will doubtless contribute to the proper determination of data for finding the practicable equivalent, in any given case.

According to the dynamical theory of gases, the average radial velocity, which is found by Prop. III, is the velocity which represents constancy of volume, in any aggregation of a homogeneous elastic fluid.

We have already seen (I) that *dynamic* constancy of pressure produces the velocity of circular revolution; and by the principles of thermodynamics, specific heat varies as *vis viva*. Prop. IV, therefore, furnishes an exact mathematical accordance with the empirical ratio between heat of constant volume and heat of constant pressure,† the representative of constant pressure in a perpetual radial oscillation, being the radius of mean excursion; the representative of constant volume, the radius of mean velocity; while the *vires vivæ*, of circular revolution due to constant pressure and of circular revolution maintaining constant volume, are inversely as their respective radii. Newcomb showed‡ that if there is actual collision of particles, they can neither be perfectly hard and spherical, nor hard and not spherical. There must, therefore, be elasticity or something analogous to elasticity, either in the gaseous particles themselves, or in their relative motions. Peirce's views, together with the fact that *vis viva* varies as the quantity of motion, (the quantity of motion, in perpetual oscillation under uniform resistance, being proportional to the average velocity), seem to involve the probability that there is no absolute collision, but the phenomena are due to simple motions, and are independent of the nature of the particles themselves. The *vis viva* of constant circular velocity varying inversely as radius, the product of such *vis viva* by radius, for any given central force, is a constant quantity.§

Prop. V may perhaps prove suggestive, in the study of the mechanical differences between fluidity and solidity, and of the laws of chemical combination, as well as in the elucidation of the nebular hypothesis, of which some illustrations are given below. I have already shown‖ that Alexander's planetary ratio, 1.8, which is the $\frac{5}{3}$ power of the thermodynamic ratio 1.4232, appears in the explosive energy of H_2O, and that this energy is directly traceable to the limiting velocity of synchronous rotation and revolution¶ under the same primitive impulses (I, III).

If the central force f, in Prop. VI, is solar gravity, the limiting velocity $(n \pi \sqrt{fr})$ of uniform impulses which account for solar and planetary

* *loc. cit.*, Feb. 4, 1851.

† The principal estimates of $\frac{c}{c'}$ are: Guthrie, Regnault, 1.41; Masson, 1.419; Tyndall, 1.421.

‡ *loc. cit.*, 1861.

§ The various *vires vivæ* considered in the present paper are:

v. v. of interior nucleal rotation in an unchanging nucleus, varying as r^2;

v. v. at any point within a homogeneously expanding or contracting nucleus, varying inversely as r^2;

v. v. of free revolution communicable by interior nucleal rotation in an unvarying nucleus, varying as r;

v. v. of free revolution due to fall towards, or repulsion from centre, varying inversely as r.

‖ Proc. S. P. A., xii, 394.

¶ Ib., and xiii, 142.

aggregation, solar rotation, and planetary revolution, is *the velocity of light.** For, Sun's distance from Earth being 214.86 solar radii, the velocity of light $= \frac{214.86r}{497.825} = .4316r$ per second. Equating this value with $n \pi \sqrt{fr}$ and substituting the value of $\sqrt{fr} = \sqrt{gr} = 2 \pi r \times 214.86 \div (365.2564 \times 86400) = .000627049r$ per second, we find $n = 219.0894$, and $v_2 = \frac{\sqrt{fr}}{n} = .0000028622\, r$ per second. On comparing this result with the estimates from observation of Sun-spots, we find it is about $\frac{2}{5}$ of one per cent. greater than Schwabe's estimate (.0000028511), while it is about $\frac{1}{4}$ of one per cent. less than the average of Spörer's greatest (.0000029533, in lat. 1° 45′) and least estimates (.0000027863, in lat. 24° 38′). All other estimates are embraced within Spörer's limits, and most of them agree very nearly with Schwabe's. Such closeness of exemplification in gravitating action, may well encourage a search for others in chemical association and dissociation.

Inasmuch as Prop. VII is a corollary of Prop. VI, it may be practically illustrated by the same examples. It has, however, an additional interest, by showing that the radial impulses continue their efficiency, within the rotating mass, until they are able to proceed, without interruption, in the same direction in which they entered the system. It also lends confirmation to the above values of n and v_2.

Prop. VIII accounts for the coefficients of planetary distances, in the abscissas of my Stellar-solar parabola† of which I offer a new form in my remarks on Prop. X.

The various planetary harmonies which have been pointed out by Titius, Bode, Alexander, Peirce, Kirkwood, and myself, furnish abundant *a posteriori* corroboration of Prop. IX.

Obermayer's Law (Trans. Vienna Acad.; quoted in *Nature,* June 1, 1876, p. 119), that the friction-coefficient of permanent gases varies as the $\frac{3}{4}$ power of the coercible gases and as the absolute temperature, seems to fall under Prop. X, as a particular case of the nucleal and dissociating influences of central force. The planets furnish various other cases, under the same proposition, illustrating Peirce's views as to the constancy of the quantity of motion, or *vis viva,* in a system, Oliver's æsthetic theorem, my planetary pendulums,‡ and nearly all of the postulates.

The time of terrestrial rotation being 86400 seconds, and the time of free revolution at the equator ($2 \pi r \div \sqrt{gr}$) 5074 seconds, the *vis viva* of revolution is $\left\{\frac{86400}{5074}\right\}^2 = 289.68$ times the *v. v.* of rotation at the same point. If this is the *v. v.* at the limit between complete aggregation and incipient dissociation, the *v. v.* at the limit between complete dissociation and incipient aggregation (VI, VII,) is π^2 times as great. Therefore, if Earth's

* Ib. xi, 103, and subsequent papers.

† Ib., xii, 523.

‡ Proc. Soc. Phil. Amer., xiv, 612, 622, sq.

motions represent *nucleal* formative force, the velocity of rotation at the latter point should be $\frac{1}{\pi^2} \times \frac{1}{289.68} \times \frac{2\pi r}{5074}$ = .0000004331r per second.

We have already seen (V) that the solar equatorial velocity at the lower limit is $\frac{1}{\pi}$ the velocity of light, or .1374r. But in perpetual nucleal rotation the communicable interior *v. v.* varies as the velocity, or as radius, and in circular revolution the mass $\propto$ *v. v.* $\times r$. Therefore if we refer the solar and terrestrial motions to the same nucleal radius, we get, for their respective masses, the proportion; Sun : Earth : : .1374 : .0000004331 : : 317,500 : 1. This represents a solar parallax of 8″.924 and a distance of 91,600,000 miles.

Applying a similar test to Jupiter's motions, we find no evidence of nucleal formative action, but a close approximation to atmospheric force synchronous with the Earth-aggregating nucleal force.* The identity of value in the two forces may be shown as follows :

The velocity of revolution, at Sun's equator, is $\frac{2\pi \times 91,600,000}{214.86 \times 10020.24}$ = 267.3265 miles per second. If this is the fundamental formative velocity for Earth and Jupiter, Earth's time of semi-rotation (VII) should be $\frac{267.3265 \times 5280}{32.08}$ = 43998.86 sec.; and Jupiter's, $\frac{11.08^2}{302.98} \times$ 43998.86 = 17826 sec.; representing a Terrestrial day of 24 h. 26 m. 37.72 sec., and a Jovian day of 9 h. 54 m. 12 sec. Earth's rotation-velocity, therefore, appears to have been accelerated about 2 per cent., and Jupiter's retarded about $\frac{1}{3}$ of 1 per cent. (according to Herschel's estimate of 9 h. 56 m.), since the "Beginning," when the Creative Word simultaneously established their nucleal foundations. Similar relations would hold if the Jovian and Telluric rings were formed at the same time, and the terrestrial nucleus did not appear till the inner ring had been slightly condensed.

If we assume Earth's present limiting velocity (VII) as the indicator of aggregating force, we have $n\ \pi \sqrt{fr}$ = 43200 $\times$ 32.08 ft. = 262.47 m. per sec. The solar modulus of light,† at the principal centre of gravity of our system (*c. g.* ☉ and ♃), is 505400 solar radii. Therefore the nucleal-formative $v = \sqrt{505400} \times$ the atmospheric v, and when the nucleal v was the v of light (.4316r), the atmospheric v = .0006071r = 262.47 m. This would make Sun's distance (214.86r) = 92,891,300 m.

Since the velocity of primitive nucleal rotation varied within the nucleus as r, communicating a similarly varying *vis viva* to the shrinking nucleus, the above indicator gives, as the theoretical time of solar rotation, 365.256 $\div \sqrt{214.56}$=24.912dy., which implies an equatorial velocity of .0000029192r per sec. Astronomical observations, and Props. VI, VII, and X, therefore,

*Hence, perhaps, the similar density of Sun and Jupiter.

†$v = \sqrt{2fr}$; .·. r, or *modulus*, = $v^2 \div 2f$. This involves Alexander's postulate of æthereal inertia.

furnish the following approximations to the rotation velocity, the unit being one ten-billionth of Sun's radius:

Spörer, maximum.	Nebular action.	Light action.	Schwabe.	Spörer, minim.
29533	29192	28622	28511	27863

According to the same indicator, the present height of possible solar atmosphere, or the radius of equal centripetal and centrifugal solar force should be $(214.86)^{\frac{2}{3}}\, r = 35.873\, r$; the time of planetary revolution at Sun, $1yr \div (214.86)^{\frac{3}{2}} = 10020.242$ sec.; the present limit to the velocity of constant pressure, $2\pi r \div 10020.242 = .000627049\, r$ per s; the present solar modulus of light, $(.4316 \div .000627049)^2 r = 473755.65\, r$; the present modulus atmosphere, $35.873 \times 473755.65 = 16995141 r$.

If we locate the Stellar-solar paraboloid by a vertex at $\frac{1}{\pi}$ solar radius, (the primitive locus of complete association), and an abscissa ($x_{35.873} = 35.873 \times$ modulus) at the present surface of modulus-atmosphere, the general equation, $x_n = \xi\, \eta^{\pm n}\, \zeta^{n^2}$, furnishes 9 abscissas between Sun and Mercury, and 9 between Neptune and *a* Centauri, the central 9 representing points of dissociating planetary velocity (VIII), as will be seen by the following table of secular elements :

n	Log.	x_n	Perihelion.	Mean.	Aphelion.
9.873	$\frac{3}{4}$ ☿	1.6774	1.6806	1.7950	1.8855
10.873	$\frac{1}{2}$ ♀	1.9099	1.8586	1.8905	1.9200
11.873	$\frac{2}{3}$ ⊕	2.1541	2.1256	2.1561	2.1845
12.873	$\frac{3}{4}$ ♂	2.4102	2.3248	2 3901	2.4468
13.873	$\frac{4}{5}$ Ast.	2.6779			
14.873	$\frac{5}{6}$ ♃	2.9575	2.9420	2.9692	2.9949
15.873	$\frac{6}{7}$ ♄	3.2488	3.2064	3.2447	3.2799
16.873	$\frac{7}{8}$ ♅	3.5519	3.5217	3.5571	3.5897
17.873	$\frac{8}{7}$ ♆	3.8668	3.8614	3.8678	3.8740
36.873	*a* Cen.	7.6622	7.6571		7.6860

The tabular logarithms for *a* Centauri (7.6571 and 7.6860) represent, respectively, the least and greatest estimates of its distance that have been made. The values of the secular planetary perihelia and aphelia are taken from Stockwell's "Memoir on the Secular Variations of the Elements of the Orbits of the Eight Principal Planets" (Smithsonian Contributions, 232, pp. 37, 38).

It is of course impossible, under the present limitations of our knowledge, to determine to what slight extent the foregoing calculations should be modified, by allowances for orbital eccentricity and other considerations. It seems likely, however, that the parabolic abscissas (X) are the fittest representatives of primitive nebular condensation and retardation, while the forces of constant pressure within the primitive nucleus furnish the closest approximation to the ultimate light-activity. If this opinion should be confirmed by the discussions of the late transit of Venus, the accuracy of my chemical estimate of Sun's mass and distance* will be likewise confirmed.

The generalization of Faraday, as corroborated by the measurements of Weber, Kohlrausch, and Clerk Maxwell, is thus extended to include gravitation, as well as electrostatic and electrodynamic action, in the same category of central force with light (VI), by means of an identical limiting *vis viva*. The simple mathematical correlations make the generalization still broader, so as to embrace heat (IV), chemism, cosmical and molecular aggregation, dissociation, rotation and revolution (V–X), and all central forces (I–X).

The Flow of Water Through an Opening in a Pierced Plate.

By Robert Briggs.

(*Read before the American Philosophical Society, November* 3d, 1876.)

The consideration of the subject of the *vena contracta*, or section of a vein of water emerging from an orifice under certain conditions, is made a portion of the proceedings of the Philosophical Society of Glasgow, and appears in their volume X, page 145 et seq. Four papers are published, the first of which is an extract from a letter of William Froude, Esq., C. E., F. R. S., to Sir William Thomson, dated Cheston Cross, Torquay, 20th December, 1875. Mr. Froude is quoted† * * * * "One result I have tried came out well :—The discharge through an introverted cylinder [tube] with keen edge. Here, by theory, the section of the jet ought to be exactly half of the aperture. For the conservation of stream line energy obliges the velocity to be that due to the head, while the conservation of momentum requires that the pressure on the aperture (which is here the sole operative pressure setting in the ultimate direction of the velocity generated) is only sufficient to create as much momentum, say, per second as will be resident in the length delivered per second, of a column of discharge, of half the sectional area of the aperture, if its velocity is that due to the head.

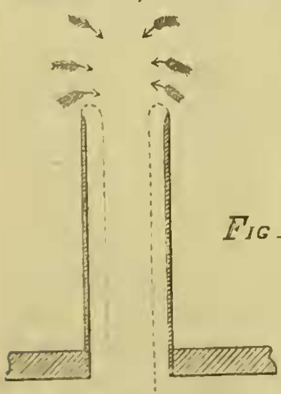

Fig 1

"The cylinder was quite smooth outside and the edge quite keen. The area ratio came out 0.503, 0.502, &c., instead of 0.500, and the little excess was obliterated, if the head was counted, to about one-fourth the diameter below the edge ; as indeed it ought to be (I won't swear to the exact figure

* Proc. S. P. A., xii, xiii, *loc. cit.*

† The entire article is quoted, the hiatus indicated by asterisks exist in the published Proceedings of the Glasgow Society.